ELLIPTIK TIPDAGI XUSUSIY HOSILALI TENGLAMALAR UCHUN QO‘YILGAN CHEGARAVIY MASALALARNI CHEKLI AYIRMALAR METODI BILAN YECHISH

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Annotatsiya: Tezisda chiziqsiz elliptik tipdagi xususiy hosilali tenglamalar uchun qo‘yilgan chegaraviy masalalarni chekli ayirmalar usuli bilan yechish algoritmi o‘rganiladi.

Kalit so‘zlar: tashqi normal, yopiq soha, elliptik, global yechim, uzluksizlik, chegaraviy tugun nuqta, umumiy yechim.

$$L[u(x,y)] = a(x,y)\frac{\partial^2 u}{\partial x^2} + 2b(x,y)\frac{\partial^2 u}{\partial x \partial y} + c(x,y)\frac{\partial^2 u}{\partial y^2} = f(x,y) \qquad (1)$$

tenglamasi berilgan bo‘lsin $(x,y) \leftarrow D$

Agar

$$\delta(x,y) = b^2(x,y) - a(x,y) - c(x,y) \quad \text{bo‘lsa}$$

(1)tenglama elliptik tipdagi tenglama deb ataladi, bunda $(x,y) \leftarrow D$. Odatda elliptik tipdagi tenglama uchun quyidagi masalalar qo‘yiladi:

1) I turdagi chegaraviy masala:

$$\begin{cases} L[n] = f & (1) \\ u|_G = Y(x,y) & (2) \end{cases}$$

bunda $Y(x,y)$ berilgan funksiya bo‘lib, $(x,y) \in G \subset D$ (G-bu D ning chegarasi. Bu masala Dirixle masalasi deb ataladi.

2) II turdagi chegaraviy masala:

$$\begin{cases} L[n] = f, & (1) \\ \frac{\partial u}{\partial n} = \varphi(x,y) & (3) \end{cases}$$

$\varphi(x,y)$-berilgan funksiya, $\frac{\partial u}{\partial n}$-tashqi normal bo‘yicha olingan hosila

3) III turdagi chegaraviy masala:

$$\begin{cases} L[n] = f & (1) \\ [\alpha\frac{\partial u}{\partial n} + \beta u]\Big|_G = \varphi, & (4) \end{cases}$$

bunda α, β, φ berilgan funksiyalar, $(x,y) \in G$

Bu masala aralash masala deb ataladi. Bu masalalarda D sohada (1) tenglamani qanoatlantiradigan, D sohaning G chegarasida 3 ta chegaraviy (2),(3),(4) shartlarning birini qanoatlantiradigan $u(x,y)$ funksiyasini topish talab qilinadi. Biz D sohani chegaralangan soha va uning chegarasi G esa uzluksiz chiziqdan iborat deb faraz qilamiz. Endi Puasson tenglamasi uchun Drixle masalasini yechishga ayirmalar sxemasini tuzishga o‘tamiz.

$$L[u(x,y)] = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y), \ (x,y) \in D \quad (5)$$

berilgan bo‘lsin. Bu tenglama $\overline{D} = D + G$ yopiq sohada yechimga ega deb faraz qilamiz va $\frac{\partial^4 u}{\partial x^4}, \frac{\partial^4 u}{\partial y^4}$- uzluksiz deb hisoblaymiz.

Oldin to‘g‘ri burchakli to‘r yasaymiz

$$x = ih, \ \ i = 0, \pm1, \pm2 \ldots$$
$$y = kl, \ \ k = 0, \pm1, \pm2; \ \ h > 0, \ l > 0 \quad (6)$$

turning OX va OY o‘qlari bo‘yichamos qadamlari. $\overline{D} = D + G$ yopiq sohasiga tegishli bo‘lgan barcha xossalarni $\overline{Dh} = Dh + Gh$ sohasiga ham tegishli bo‘ladi deb faraz qilamiz.

Ichki tugunlarida “0” chegaraviy tugunlarida “x” (5) chegaraviy tenglamani chekli ayirmalar bilan almashtirish uchun oldin ushbu tenglamaga o‘tamiz.

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{(xi,yk)} + \left.\frac{\partial^2 u}{\partial y^2}\right|_{(xi,yk)} = f(xi,yk) \qquad (7)$$

dagi mos hosilalarida chekli ayirma nisbatlariga almashtiramiz. Unda

$$L[u(xi,yk)] = \frac{u(xi+1,yk)-2u(xi,yk)+u(xi-1,yk)}{h^2} - \frac{h^2}{l^2}\left.\frac{\partial^4 u}{\partial x^4}\right|_{(xi,yk)} +$$
$$\frac{u(xi,yk+1)-2u(xi,yk)+u(xi,yk-1)}{h^2} - \frac{l^2}{l^2}\left.\frac{\partial^2 u}{\partial yk}\right|_{(xi,yk^{(1)})} = f(xi,yk) \qquad (8)$$

$$(xi,yk) \in Dh \qquad x_{i-1} < x_i^{(1)} < x_{i+1,}\ y_{k-1} < y_k^{(1)} < y_{k+1}$$

h va l ning kichik qiymatlarida h^2 va l^2 ko‘paytmalari kichik ifoda bo‘ladi va (8) dagi ifoda $L_k^{(1)}[n^k] = f^k$ ayirmali sxemaga ega bo‘lamiz, bunda

$$L_k^{(1)}[n^k] = \frac{u_{i+1k}-2u_{ik}-u_i-lh}{h^2} + \frac{u_{k+1}-2u_k-l}{h^2} \qquad (9)$$

$$f^k = f(xi,yk), \qquad (xi,yk) \in Dh, \qquad uik \approx u(xi,yk)$$

Agar (9) va (8) dagi tashlab yuborilgan hadlarni hisobga olsak, unda

$$L_k^{(1)}[u_k] = f^k + \partial f^k \qquad (10)$$

Bunda $f^k = \frac{l^2}{i^2}\left.\frac{\partial^4 u}{\partial x^4}\right|_{(xi,yk)} + \frac{h^2}{i^2}\left.\frac{\partial^4 u}{\partial y^4}\right|_{(xi,yk)}$. Bu yerdan

$$\left|\partial f^k\right| \le Mh^2 \qquad (11)$$

ekani kelib chiqadi, bu yerda $l = \alpha h$, $\alpha > 0$. Bu esa (9) ayirmali sxema (5) differensial tenglamaning $u(x,y)$ yechimni $O(h^2)$ xatoligi bilan yaqinlashtirishini anglatadi. Chegaraviy shartni G_A chegarada yaqinlashtirish:

$\beta = (xi, yk) \in G_A$ bo‘lsin, $M \in G$ uchun $M = M(xi, yk)$, $A \in Dh$ uchun $A = A(xi, yk)$ u holda chegaraviy shart:

$$u_k \approx y(xi, yk) \tag{12}$$

dan

$$y(xi, yk) = u(xi - \delta, yk) = u(xi, yk) - \frac{\delta}{i}\frac{\partial u}{\partial x}(xi, yk)$$

Bunda

$$y(xi, yk) - u_k = -\frac{\delta}{i}\frac{\partial u}{\partial x}(xi, yk) \tag{13}$$

Agar $\delta = \alpha h$- to‘g‘ri bo‘ladigan $\alpha > 0$ soni mavjud bo‘lsa,u holda (12) formula xatoligi (13) formula bilan aniqlanadi.

$B(xi, yk) \in G_A$ nuqtasida chegaraviy shartni aniqroq yaqinlashtirishi uchun $A(xi - h, yk) \in D_k$ nuqtasidan foydalanish kerak. Bundan

$u(m) = u(xi - h, yk) = u(B) + \frac{\delta}{1!}\frac{\partial u}{\partial x}\Big|_B + \frac{\delta^2}{2!}\frac{\partial^2 u}{\partial x^2}\Big|_{\overline{B}}$ bu yerda $M < \overline{B} < B$

Xuddi shunday $u(A) = u(xi + h, yk) = u(B) + \frac{h}{1!}\frac{\partial u}{\partial x}\Big|_B + h^2 2! \frac{\partial^2 u}{\partial x^2}\Big|_{\overline{A}}$,

bunda $B < \overline{A} < A$

Demak, $u(m)h + \delta u(A) = (h + \delta)u(B) + \frac{\delta^2 h + \delta^2 H}{2!}[\frac{\partial^2 u}{\partial x^2}\Big|_{\overline{B}} + \frac{\partial^2 u}{\partial x^2}\Big|_{\overline{A}}$ yoki $u(m)h + \delta u(A) = (h + \delta)u(B) + O(h^2)$ ya’ni $u(B) = \frac{hu(m)+\delta u(A)}{h+\delta} + O(h^2)$ bu tenglamadan $O(h^2)$ ni tanlab

$$u(xi, yk) = \frac{hu(xi-h,yk)+\delta u(xi,yk)}{h+\delta} \tag{14}$$

turdagi taqribiy formula G_A ga tegishli bo‘lgan ixtiyoriy tugun nuqta uchun yechiladi. Shunday qilib (9) va (14) formulalaridan foydalanib, (5) va (6) chegaraviy masalalarni $O(h^2 + l)$ xatolik bilan aniqlashining ushbu ayirmali sxemasiga ega bo‘lamiz.

$$L_k^{(1)}[u^k] = f^k \tag{15}$$

bunda

$$L_k{}^{(1)}[u^k] = \{\frac{u_{i+1k} - 2u_{ik} + u_{ik+1}}{h^2} + \frac{u_{i+1k} + 2u_{ik} + u_{ik+1}}{l^2}\}\ (ih, kl) \in Dh,$$
$$u(ih, kl) \in Fh$$

$$F^k = \begin{cases} f(xi, yk), (ih, kl) \in Dh \\ u(xi, yk),\ (ih, kl) \in G_k \end{cases} \quad (17)$$

endi $[\alpha(x,y)\frac{\partial u}{\partial n} + \beta(x,y)u(x,y)]$ umumiy ko‘rinishdagi chegaraviy shartniyaqinlashtirish masalasini qaraymiz.

(17) da $\alpha^2 + \beta^2 > 0$. $O \in G_A$ chegaraviy tugun bo‘lsin. Xamisha shu O tugundan chiqarilgan 2ta l_1 va l_2 nurlardan 2 ta nuqtasi topiladi. U holda

$$\frac{\partial u}{\partial n} = \frac{\partial u}{\partial x}\cos(\alpha_0) + \frac{\partial u}{\partial y}\cos(\alpha) \quad (18)$$

Agar chekli ayirmalariga o‘tsak:

$$\left.\frac{\partial u}{\partial x}\right|_{(1,0)} = \frac{u_1 - u_0}{l_1} + O(u);\ \left.\frac{\partial u}{\partial y}\right|_{(2,0)} = \frac{u_2 - u_0}{l_2} + O(h)$$

Demak, $\left.\frac{\partial u}{\partial n}\right|_{(1,2)} = \frac{u_1 - u_0}{l_1}\cos(\alpha_0) + \frac{u_2 - u_0}{l_2}\sin(\alpha_0) + O(h)$

$$\alpha_0[\frac{u_1 - u_0}{l_1}\cos(\alpha_0) + \frac{u_2 - u_0}{l_2}\sin(\alpha_0)] + \beta_0 u_0 = \varphi_0 \quad (19)$$

$\alpha_0, \beta_0, \varphi_0 - \alpha, \beta, \varphi$ – mos funksiyalarning 0 chegaraviy nuqtalariga yaqin bo‘ladigan qiymatlari bo‘ladi. Bunday yaqinlashtirishdagi xatolik O(h) bo‘ladi.

Foydalanilgan adabiyotlar

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