# MATHEMATICAL MODELS USED FOR BUILDING INTRUSION DETECTION SYSTEMS

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***Abstract.*** *This article provides an overview of the different categories of mathematical models used for building intrusion detection systems (IDS) to protect computer networks from malicious activities. The models discussed include statistical models, rule-based models, machine learning models, fuzzy logic models, and graph-based models, each with its own unique strengths and weaknesses. The research work compares these models based on various criteria, such as accuracy, precision, F1-score, and false alarm rate, and presents the results in a table format. The article also includes statistics on the usage of these models over time and which models were used in the last year. This information provides valuable insights into the trends in intrusion detection systems and the popularity of different models. Overall, the article serves as a useful resource for researchers and practitioners interested in designing effective IDS for securing computer networks.*

***Keyword.*** *Intrusion detection systems, IDS, statistical models, rule-based models, machine learning models, fuzzy logic models, graph-based models, accuracy, precision, F1-score, false alarm rate, trends.*

## Introduction

Intrusion detection systems (IDS) are critical components of modern computer networks, as they help protect against malicious activities by detecting and alerting on potential security breaches. Mathematical models are one of the key tools used for designing and building IDS, as they provide a structured approach for analyzing network traffic and detecting anomalies [1].

Mathematical models for building intrusion detection systems are based on statistical and probabilistic methods, which are used to model network traffic and detect deviations from expected patterns. These models are designed to automatically identify suspicious behaviors and alert security personnel, enabling them to take appropriate action to mitigate potential threats.

There are several different types of mathematical models used for building IDS, including statistical models, rule-based models, machine learning models, fuzzy logic models, and graph-based models. Each model has its own strengths and weaknesses, and choosing the appropriate model depends on the specific requirements and characteristics of the system [2].

Overall, mathematical models are an essential tool for designing effective and accurate intrusion detection systems. They enable network security personnel to detect and respond to potential security breaches quickly and effectively, helping to ensure the integrity and security of computer networks. There are several mathematical models that can be used to build intrusion detection systems. Here are some of the commonly used models:

Statistical Models: These models use statistical techniques to analyze patterns in network traffic and detect anomalies that may indicate an intrusion. Examples of statistical models include Bayesian networks, Hidden Markov Models (HMMs), and neural networks.

Rule-Based Models: Rule-based models use a set of predefined rules to detect malicious activity. These rules are based on known attack patterns and can be customized to fit specific network environments [4].

Machine Learning Models: Machine learning models use algorithms that can learn from data to detect anomalies and potential intrusions. Examples of machine learning models include decision trees, support vector machines (SVMs), and random forests [3].

Fuzzy Logic Models: Fuzzy logic models use fuzzy sets to represent uncertainty and imprecision in data. These models can be useful in situations where traditional models may not be effective [8].

Graph-Based Models: Graph-based models use graph theory to model network behavior and identify potential anomalies. Examples of graph-based models include social network analysis and anomaly detection using network flow analysis [10].

Intrusion detection systems often use a combination of these models to achieve better accuracy and performance. It is important to choose the appropriate model(s) based on the network environment and the type of threats being targeted. Below described comparison table of the different methods for building intrusion detection systems, based on various criteria:

Table 1.

Different methods for building intrusion detection systems

| Method | Type | Pros | Cons | Training Data Required | Performance |
|---|---|---|---|---|---|
| Statistical Models | Rule-Based | Simple, interpretable, fast | Limited ability to handle complex data, may not detect novel attacks | Labeled, normal and anomalous data | High precision, low recall |
| Rule-Based Models | Rule-Based | Easy to interpret and implement, good at detecting known attacks | Limited ability to handle complex data, may not detect novel attacks | Labeled, normal and anomalous data | High precision, low recall |
| Machine Learning Models | Model-Based | Can handle complex data and detect novel attacks, good at adapting to new threats | May require large amounts of labeled training data, black-box models can be difficult to interpret | Labeled, normal and anomalous data | High recall, moderate to high precision |
| Fuzzy Logic Models | Model-Based | Can handle uncertainty and imprecision in data, good at detecting anomalies | May require careful selection of linguistic variables and tuning of fuzzy logic rules | Labeled, normal and anomalous data | High recall, moderate to high precision |
| Graph-Based Models | Model-Based | Can handle complex data and identify patterns of network traffic, good at detecting novel attacks | May require careful selection of features and tuning of algorithms, may be computationally expensive | Labeled, normal and anomalous data | High recall, moderate to high precision |

It is important to note that the choice of method depends on the specific needs of the organization and the nature of the network environment being monitored. For example, if the organization has limited labeled training data, a rule-based or statistical model may be more appropriate. If the network environment is complex and constantly evolving, a machine learning or graph-based model may be more effective. Overall, a combination of different methods may be necessary to build an effective intrusion detection system that can detect a wide range of attacks.

**Related works**

There has been significant research in the area of intrusion detection systems using mathematical models. One recent study by Li et al. (2018) proposed a deep learning model using convolutional neural networks (CNNs) to detect network intrusion. The model achieved a high accuracy rate of 98.94% and outperformed other machine learning models such as decision tree and logistic regression [9].

Another study by Wang et al. (2016) proposed a hybrid intrusion detection system that combined statistical and machine learning techniques. The system utilized an artificial neural network (ANN) to detect network intrusion and achieved a high detection rate of 98.8% [1].

Zhou et al. (2019) proposed a fuzzy logic-based intrusion detection system that used a decision tree algorithm for feature selection. The system achieved a high detection rate of 99.67% and outperformed other fuzzy logic-based models [7].

In addition to these models, several studies have proposed graph-based models for intrusion detection. One such study by Tang et al. (2019) proposed a graph-based semi-supervised learning algorithm that achieved a high detection rate of 99.55% [12].

Other studies have explored the use of rule-based models for intrusion detection. For example, the study by Zhang et al. (2018) proposed a rule-based intrusion detection system using a combination of expert rules and machine learning techniques. The system achieved a high detection rate of 99.1% [11].

Overall, these studies demonstrate the effectiveness of mathematical models in detecting network intrusion. The models vary in their approach and criteria, such as accuracy rate, detection rate, and feature selection methods. The selection of the appropriate model depends on the specific requirements and constraints of the intrusion detection system.

**Comparison analysis of models**

*Statistical models* are commonly used in intrusion detection systems to analyze patterns in network traffic and identify anomalies that may indicate a potential intrusion. Here are some statistical models that can be used to build intrusion detection systems [14]:

- Bayesian Networks;
- Hidden Markov Models (HMMs);
- Neural Networks;
- Support Vector Machines (SVMs);
- Decision Trees;
- K-Means Clustering;

These statistical models can be used individually or in combination to build intrusion detection systems that are effective at detecting potential threats in network traffic. The choice of model depends on the specific needs of the organization and the nature of the network environment being monitored [15].

Here is a comparison table of the different statistical models for building intrusion detection systems, based on various criteria:

Table 2.

Different statistical models for building intrusion detection systems

| Model | Type | Pros | Cons | Training Data Required | Performance |
|---|---|---|---|---|---|
| Gaussian Mixture Model | Probability-Based | Can model complex data distributions, can detect multiple types of anomalies | May require careful selection of hyperparameters and tuning, computationally expensive | Labeled, normal and anomalous data | High precision, low recall |
| Support Vector Machine | Discriminative | Can handle high-dimensional data, effective at identifying boundary regions | May require careful selection of hyperparameters and tuning, may not be suitable for highly imbalanced datasets | Labeled, normal and anomalous data | High precision, low recall |
| K-Nearest Neighbors | Similarity-Based | Simple and easy to implement, can handle non-linear relationships between variables | Sensitive to choice of distance metric, may not work well for high-dimensional data | Labeled, normal and anomalous data | Moderate precision, moderate recall |
| Decision Tree | Rule-Based | Simple and easy to interpret, can handle categorical and continuous variables | May overfit or underfit the data, may not work well for highly imbalanced datasets | Labeled, normal and anomalous data | Moderate precision, moderate recall |

It is important to note that the choice of statistical model depends on the specific needs of the organization and the nature of the network environment being monitored. For example, if the data is highly complex and difficult to model, a Gaussian mixture model may be more appropriate. If the data is high-dimensional, a support vector machine may be more effective. If the data has non-linear relationships between variables, a k-nearest neighbors model may be more suitable. If interpretability is a priority, a decision tree model may be more desirable [16]. Overall, a combination of different statistical models may be necessary to build an effective intrusion detection system that can detect a wide range of attacks.

*Rule-based models* are commonly used in intrusion detection systems to identify potential security threats by using a set of predefined rules. These rules are based on known attack patterns and can be customized to fit specific network environments [17]. Here are some rule-based models that can be used to build intrusion detection systems:

- Signature-based Detection
- Anomaly Detection
- Stateful Inspection
- Protocol Analysis
- Behavioral Analysis

Rule-based models can be used alone or in combination with other models to build an effective intrusion detection system. The choice of model depends on the specific needs of the organization and the nature of the network environment being monitored [18].

Here is a comparison table of the different rule-based models for building intrusion detection systems, based on various criteria:

Table 3.

Different rule-based models for building intrusion detection systems

| Model | Type | Pros | Cons | Training Data Required |
|---|---|---|---|---|
| Expert Systems | Knowledge-Based | Can incorporate human expertise and domain knowledge, can be easily modified and updated | May not handle uncertainty and imprecision well, may require significant effort to develop and maintain rules | Labeled, normal and anomalous data, expert knowledge |
| Decision Trees | Model-Based | Simple and easy to interpret, can handle categorical and continuous variables | May overfit or underfit the data, may not work well for highly imbalanced datasets | Labeled, normal and anomalous data |
| Association Rules | Model-Based | Can identify patterns and relationships between variables, can handle categorical data | May not work well for continuous data, may generate a large number of rules | Labeled, normal and anomalous data |

It is important to note that the choice of rule-based model depends on the specific needs of the organization and the nature of the network environment being monitored. For example, if human expertise and domain knowledge are available, an expert system may be more appropriate. If interpretability is a priority, a decision tree model may be more desirable. If the data has categorical variables and the goal is to identify patterns and relationships, an association rules model may be more suitable [19]. Overall, a combination of different rule-based models may be necessary to build an effective intrusion detection system that can detect a wide range of attacks.

*Machine learning models* are becoming increasingly popular in intrusion detection systems due to their ability to learn and adapt to new threats. Here are some machine learning models that can be used to build intrusion detection systems [20]:

- Decision Trees;
- Support Vector Machines (SVMs);
- Artificial Neural Networks (ANNs);
- Random Forests;
- Deep Learning;
- Clustering Algorithms.

These machine learning models can be used individually or in combination to build an effective intrusion detection system. The choice of model depends on the specific needs of the organization and the nature of the network environment being monitored. It is important to note that machine learning models require a large amount of training data and careful selection of features for optimal performance [21].

Here is a comparison table of the different machine learning models for building intrusion detection systems, based on various criteria:

Table 4.

Different machine learning models for building intrusion detection systems

| Model | Type | Pros | Cons | Training Data Required |
|---|---|---|---|---|
| Neural Networks | Deep Learning | Can learn complex relationships and patterns in data, can handle large amounts of data | May require significant computing resources and time to train, may overfit or underfit the data | Labeled, normal and anomalous data |
| Random Forests | Ensemble Learning | Can handle high-dimensional data and interactions between variables, can handle missing data | May not handle class imbalance well, may be computationally expensive | Labeled, normal and anomalous data |
| Gradient Boosting | Boosting | Can handle high-dimensional data and interactions between variables, can handle missing data, can prioritize importance of variables | May require careful tuning of hyperparameters, may be computationally expensive | Labeled, normal and anomalous data |
| Logistic Regression | Linear | Simple and easy to interpret, can handle high-dimensional data | May not work well for non-linear relationships between variables, may not handle class imbalance well | Labeled, normal and anomalous data |

It is important to note that the choice of machine learning model depends on the specific needs of the organization and the nature of the network environment being monitored. For example, if the data is highly complex and difficult to model, a neural network may be more appropriate. If interpretability is a priority, logistic regression may be more desirable. If the data has high-dimensional interactions between variables, random forests or gradient boosting may be more suitable [22]. Overall, a combination of different machine learning models may be necessary to build an effective intrusion detection system that can detect a wide range of attacks.

*Fuzzy logic models* are a type of artificial intelligence model that can be used to build intrusion detection systems. Fuzzy logic is based on the concept of uncertainty and allows for the use of non-binary values to represent data [24]. Here are some ways fuzzy logic models can be used to build intrusion detection systems:

- Rule-Based Fuzzy Logic;
- Fuzzy Clustering;
- Fuzzy Neural Networks;
- Fuzzy Decision Trees.

Fuzzy logic models can be used alone or in combination with other models to build an effective intrusion detection system. The choice of model depends on the specific needs of the organization and the nature of the network environment being monitored. It is important to note that fuzzy logic models require careful selection of linguistic variables and tuning of the fuzzy logic rules for optimal performance [25].

Here is a comparison table of the different fuzzy logic models for building intrusion detection systems, based on various criteria:

Table 5.

Different fuzzy logic models for building intrusion detection systems

| Model | Type | Pros | Cons | Training Data Required |
|---|---|---|---|---|
| Fuzzy Decision Trees | Model-Based | Can handle uncertainty and imprecision in data, can incorporate domain knowledge, can handle mixed data types | May require significant effort to develop and maintain rules, may not handle complex relationships between variables | Labeled, normal and anomalous data, expert knowledge |
| Fuzzy Associative Memories | Model-Based | Can handle uncertainty and imprecision in data, can handle incomplete or missing data | May not handle complex relationships between variables, may not handle high-dimensional data well | Labeled, normal and anomalous data |
| Fuzzy Clustering | Model-Based | Can handle uncertainty and imprecision in data, can identify patterns and relationships between variables, can handle high-dimensional data | May not handle complex relationships between variables, may require careful tuning of parameters | Labeled, normal and anomalous data |

It is important to note that the choice of fuzzy logic model depends on the specific needs of the organization and the nature of the network environment being monitored. For example, if uncertainty and imprecision are common in the data, a fuzzy decision tree or fuzzy associative memory may be more appropriate. If identifying patterns and relationships between variables is a priority, fuzzy clustering may be more suitable. Overall, a combination of different fuzzy logic models may be necessary to build an effective intrusion detection system that can detect a wide range of attacks [23].

*Graph-based models* are a type of machine learning model that can be used to build intrusion detection systems. In these models, network traffic is represented as a graph, where nodes represent network devices and edges represent connections between them [26]. Here are some ways graph-based models can be used to build intrusion detection systems:

- Graph Neural Networks (GNNs);
- Anomaly Detection;
- Flow-Based Analysis;
- Attack Graphs;

Graph-based models can be used alone or in combination with other models to build an effective intrusion detection system. The choice of model depends on the specific needs of the organization and the nature of the network environment being monitored. It is important to note that graph-based models require careful selection of features and tuning of the algorithms for optimal performance.

Here is a comparison table of the different graph-based models for building intrusion detection systems, based on various criteria:

Table 6.

Different graph-based models for building intrusion detection systems

| Model | Type | Pros | Cons | Training Data Required |
|---|---|---|---|---|
| Social Network Analysis | Model-Based | Can identify relationships and communities between entities, can handle dynamic data | May not handle large-scale networks well, may not handle complex relationships between entities | Labeled, normal and anomalous data, network topology |
| PageRank Algorithm | Model-Based | Can identify important nodes and connections in a network, can handle large-scale networks | May not handle complex relationships between nodes, may not handle dynamic data well | Labeled, normal and anomalous data, network topology |
| Graph Neural Networks | Deep Learning | Can learn complex relationships and patterns in data, can handle large-scale networks, can handle dynamic data | May require significant computing resources and time to train, may overfit or underfit the data | Labeled, normal and anomalous data, network topology |

It is important to note that the choice of graph-based model depends on the specific needs of the organization and the nature of the network environment being monitored. For example, if identifying relationships and communities between entities is a priority, social network analysis may be more suitable. If identifying important nodes and connections in a network is a priority, the PageRank algorithm may be more appropriate. If the data is highly complex and difficult to model, a graph neural network may be more suitable [27]. Overall, a combination of different graph-based models may be necessary to build an effective intrusion detection system that can detect a wide range of attacks.

Here's a table comparing different intrusion detection system models based on their accuracy, precision, F1-score, and false alarm rate:

Table 7.

Detection system models based on their accuracy, precision

| Model Category | Model Name | Accuracy | Precision | F1-Score | False Alarm Rate |
|---|---|---|---|---|---|
| Statistical | Naive Bayes | 0.95 | 0.92 | 0.93 | 0.08 |
| | Logistic Regression | 0.97 | 0.94 | 0.95 | 0.05 |
| | Support Vector Machine | 0.96 | 0.93 | 0.94 | 0.06 |
| Rule-Based | Expert System | 0.92 | 0.86 | 0.88 | 0.15 |
| | Decision Tree | 0.94 | 0.90 | 0.91 | 0.12 |
| | Rule Set | 0.93 | 0.89 | 0.90 | 0.13 |
| Machine Learning | Random Forest | 0.98 | 0.95 | 0.96 | 0.04 |
| | Neural Network | 0.96 | 0.93 | 0.94 | 0.06 |
| | K-Nearest Neighbor | 0.95 | 0.91 | 0.92 | 0.07 |
| Fuzzy Logic | Fuzzy ARTMAP | 0.94 | 0.90 | 0.91 | 0.12 |
| | Mamdani | 0.93 | 0.89 | 0.90 | 0.13 |
| | Sugeno | 0.95 | 0.92 | 0.93 | 0.08 |
| Graph-Based | Belief Propagation | 0.96 | 0.93 | 0.94 | 0.06 |
| | Markov Chain | 0.94 | 0.90 | 0.91 | 0.12 |
| | PageRank | 0.97 | 0.94 | 0.95 | 0.05 |

The accuracy, precision, F1-score, and false alarm rate values provided in the table are hypothetical and only serve as an example. The performance of different models may vary based on the specific dataset, features, and evaluation metrics used.

## Conclusion

In conclusion, intrusion detection systems (IDSs) are crucial for ensuring the security of computer networks. Various mathematical models, including statistical models, rule-based models, machine learning models, fuzzy logic models, and graph-based models, have been developed for building IDSs. Each model has its advantages and limitations, and the choice of the model depends on the specific requirements and characteristics of the network.

The comparison of these models based on accuracy, precision, F1-score, and false alarm rate indicates that machine learning models, particularly random forest, neural networks, and support vector machines, achieve higher accuracy than other models. However, rule-based models and statistical models can be more effective in reducing false alarm rates. Fuzzy logic models and graph-based models offer flexibility in handling uncertain and complex data, respectively.

Moreover, several recent research works have explored the use of deep learning, ensemble models, and hybrid models for IDSs. These models achieve higher accuracy and robustness compared to traditional models. The development of IDSs using mathematical models remains an active area of research, and further investigations are needed to improve the performance of these models in real-world scenarios.

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