ETALON INTEGRALLAR

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Annotatsiya: *Furye integrali uchun zarur bo‘lgan Erdey lemmasi va shu lemmaning tasdiqi isboti haqida.*

Аннотация: *О лемме Эрдей, необходимой для интеграла Фурье, и доказательстве утверждения этой леммы.*

Annotation: *About the Erdey lemma necessary for the Furye integral and the proof of confirmation of that lemma.*

Quyidagi integralni qaraymiz

$$\Phi(\lambda)=\int_0^a x^{\beta-1} f(x)e^{i\lambda x^\alpha}dx. \qquad (1)$$

1-Lemma. (Erdey lemmasi). Agar $\alpha \geq 1,\ \beta > 0,\ f(x) \in C^\infty([0,a])$ va x=a nuqtada o‘zini hosilalari bilan birgalikda nolga aylansa . U holda

$$\int_0^a x^{\beta-1}f(x)e^{i\lambda x^\alpha}dx \sim \sum_{k=0}^{\infty} a_k \lambda^{-\frac{k+\beta}{\alpha}} \qquad (\lambda \to +\infty) \qquad (2)$$

$$a_k=\frac{f^{(k)}(0)}{k!\alpha}\Gamma\left(\frac{k+\beta}{\alpha}\right)\exp\left[\frac{i\pi(k+\beta)}{2\alpha}\right]. \qquad (3)$$

Bu yoyilmani λ bo‘yicha ixtiyoriy martta differensiallash munkin.

Erdey lemmasi Furye integrali uchun shunday ro‘l o‘ynaydiki xuddi Vatson lemmasini Laplas integrallarida tutgan o‘rni kabi.

Isbot. $S = x^\alpha$ faza funksiya yagona x=0 kritik nuqtaga ega integrallash sohasida.

Birinchi navbatda $0 \leq x \leq \delta, bunda\ 0 < \delta < a\ da\ f(x) \equiv 1$ bo‘ladi. U holda Integral ostidagi funksiya (0,δ) da analitik funksiya bo‘ladi. 0<$argx < \pi/\alpha$ sektorda biz quyidagiga ega bo‘lamiz $Re(ix^\alpha) < 0$. Koshi teoremasiga ko‘ra $[0,\frac{\delta}{2}]$ kesma bo‘yicha integral $l = l_1 \cup l_2$ bo‘yicha integralga teng bunda $l_1 - kesma\ [0, e^{\frac{i\pi}{2\alpha}}\rho_0],\ l_2 - kesma\ [e^{\frac{i\pi}{2\alpha}}\rho_0, \delta/2]$. U holda

$$\Phi_\beta(\lambda)=\Phi_\beta^{(1)}(\lambda)+\Phi_\beta^{(2)}(\lambda)+\Phi_\beta^{(3)}(\lambda) \qquad (4)$$

bunda quyidagiga ega bo‘lamiz

$$\Phi_\beta^{(1)}(\lambda) = \int_0^{e^{\frac{i\pi}{2\alpha}}\rho_0} x^\beta e^{i\lambda x^\alpha}\, dx = e^{\frac{i\pi(\beta+1)}{2\alpha}} \int_0^\rho x^\beta e^{-\lambda x^\alpha} dx =$$
$$\alpha^{-1}\Gamma\left(\frac{\beta+1}{\alpha}\right) e^{\frac{i\pi(\beta+1)}{2\alpha}} \lambda^{-\frac{\beta+1}{\alpha}} + O(e^{-c\lambda}) \qquad (5)$$

Vatson lemmasiga ko‘ra . Bo‘laklab integrallab , quyidagiga ega bo‘lamiz

$$\Phi_\beta^{(2)}(\lambda) + \Phi_\beta^{(3)}(\lambda) = \frac{x^\beta \exp\,(i\lambda x^\alpha)}{i\alpha\lambda x^{\alpha-1}} \Big|_{e^{\frac{i\pi}{2\alpha}}\rho_0}^{\delta/2} -$$
$$-(i\alpha\lambda)^{-1} \int_{l_2} \exp(i\lambda x^\alpha) \left(x^{\beta-\alpha+1}\right)' dx + (i\alpha\lambda x^{\alpha-1})^{-1} x^\beta f(x)\, exp(i\lambda x^\alpha) \Big|_{\frac{\delta}{2}}^{\alpha} -$$
$$-(i\alpha\lambda)^{-1} \int_{\delta/2}^{\alpha} exp(i\lambda x^\alpha) \left(f(x) x^{\beta-\alpha+1}\right)' dx. \qquad (6)$$

Bundan o‘z-o‘zidan (6) dagi integral ostidagi postanovka $O(\lambda^{-\infty})$ tartibga ega va bundan tashqari

$$\Phi_\beta^{(2)}(\lambda) + \Phi_\beta^{(3)}(\lambda) = O(\lambda^{-\infty}) \qquad (\lambda \to +\infty) \qquad (7)$$

$\lambda \geq 0$ da l_1, l_2, l_3 kesmalar bo‘yicha $|\exp(i\lambda x^\alpha)| \leq 1$ eknaligidan va $f'(x) \epsilon C^\infty([0, a])\ va\ 0 \leq x \leq \delta\ da\ f'(x) \equiv 0$ ekanligidan foydalansak. Quyidagiga kelamiz

$$\Phi_\beta^{(2)}(\lambda) + \Phi_\beta^{(3)}(\lambda) = \frac{\alpha-\beta}{i\lambda\alpha} [\Phi_{\beta-\alpha}^{(2)}(\lambda) + \Phi_{\beta-\alpha}^{(3)}(\lambda)] + O(\lambda^{-\infty}). \qquad (8)$$

Bizdai (7) ga ko‘ra $\Phi_\beta^{(2)}(\lambda) + \Phi_\beta^{(3)}(\lambda) = O(\lambda^{-1})$ va (8) ga ko‘ra $\Phi_\beta^{(2)}(\lambda) + \Phi_\beta^{(3)}(\lambda) = O(\lambda^{-2})$ ga ega bo‘lamiz bu jarayondi davom etirib quyidagiga ega bo‘lamiz $\Phi_\beta^{(2)}(\lambda) + \Phi_\beta^{(3)}(\lambda) = O(\lambda^{-\infty})$ bu baholashdan va (4), (5) ga ko‘ra quyidagiga ega bo‘lamiz

$$\Phi_\beta(\lambda) = \alpha^{-1}\Gamma\left(\frac{\beta+1}{\alpha}\right) e^{\frac{i\pi(\beta+1)}{2\alpha}} \lambda^{-\frac{\beta+1}{\alpha}} + O(\lambda^{-\infty}) \qquad (\lambda \to +\infty) \quad (9)$$

Agar kichik x larda $f(x) \equiv 1$ bo‘lsa. Lemmani umumiy holatda Teylor Formulasiga ko‘ra isbot qilamiz

2-Lemma. Erdey lemmasidagi tasdiq o‘rinli $|\lambda| \to \infty, 0 \leq arg\lambda \leq \pi$ bo‘lganda xuddi shunday $arg\lambda$ bo‘yicha ham.

Isbot. Agar $\varepsilon \leq \theta = arg\lambda \leq \pi - \varepsilon,$ bunda $0 < \varepsilon < \pi$ unda (1) integral Vatson lemmasini shartlarini bajaradi . shuning uchun quyidagini isbotlasak yetarli (2) asimptotikasi uchun quyidagi o‘rinli $0 \leq \theta \leq \varepsilon,\ \pi - \varepsilon \leq \theta \leq \pi$ sektorlarda qayerdaki $\varepsilon > 0$ ni ixtiyoriycha kichik tanlash munkin. Agar $0 \leq \theta \leq \varepsilon$ u holda $0 \leq argx \leq \pi/2\alpha$ sektorda quyidagiga ega bo‘lamiz

$$Re(i\lambda x^\alpha) = -|\lambda||x^\alpha| \sin\,(\theta + \alpha argx) \leq 0$$

Bunda albatda $0\leq \theta + \alpha argx \leq \frac{\pi}{2\alpha} + \varepsilon$ ekanligini hisobga oldik. O‘z-o‘zidan l_1, l_2, l_3 kesmalar bo‘yicha $|\exp(i\lambda x^{\alpha})|\leq 1$ va yuqoridagi lemma isbotida qurilgan narsalar to‘liq $0 \leq \theta \leq \varepsilon$ holat uchun ko‘chiriladi. Analitik ravishda $\pi - \varepsilon \leq \theta \leq \pi$ ga ham.

Misol 1. $\lambda \to +\infty$ bo‘lganda quyidagi munosabat o‘rinli ekanligini isbotlang

$$\int_0^1 \exp(i\lambda x^3)\, dx \sim \Gamma\left(\frac{4}{3}\right) e^{\frac{i\pi}{6}} \lambda^{-\frac{1}{3}} - \sum_{k=0}^{\infty} \frac{\Gamma(k+\frac{2}{3})}{\Gamma(-\frac{1}{3})} (i\lambda)^{-k-1} e^{i\lambda}.$$

Erdey tomonidan 1-lemmani boshqacharoq isboti keltirilgan va biz uni keltiramiz sababi u bazi foydali texnik ozgarishlar ega . $\alpha \geq 1, 0 < \beta \leq 1$ bo‘lsin,

$\Phi_\beta(\lambda)$ ni quyidagicha ko‘rinishda keltiramiz :

$$\Phi_\beta(\lambda) = \int_0^{\alpha} f(x) d\left(\int_{\infty}^{x} t^{\beta-1} e^{i\lambda t^{\alpha}} dt\right)$$

va bo‘laklab integrallaymiz . bunda integral quyidagi o‘q bo‘yicha olinayabdi

$l_t: t = x + \rho e^{i\frac{\pi}{2\alpha}},\ \rho > 0, t$ kompleks tekislikdan. N marta bo‘laklab integrallab quyidagiga ega bo‘lamiz

$$\Phi_\beta(\lambda) = \sum_{n=0}^{N-1} (-1)^n f^{(n)}(0) ц_{-n-1}(0,\lambda) + R_N(\lambda),$$

$$R_N(\lambda) = (-1)^{N+1} \int_0^{\alpha} \varphi_{-N}(x,\lambda) f^N(x) dx \qquad (10)$$

Bu yerda quyidagicha aniqlangan

$$\varphi_{-n-1}(x,\lambda) = \frac{(-1)^{n+1}}{n!} \int_{l_x} (t-x)^n t^{\beta-1} \exp(i\lambda t^{\alpha})\, dt \qquad (11)$$

(10) dagi yig‘indi boshlang‘ich N elementini baholshga imkon beradi. Bundan quyidagi natijaga kelamiz

$$|R_N(\lambda)| \leq C_N \lambda^{-\frac{N}{\alpha}} \int_0^{\alpha} x^{\beta-1} dx = C_N' \lambda^{-\frac{N}{\alpha}} \qquad (12)$$

Bunda N ixtiyoriy ekanligidan yuqoridagi munosabat isbot bo‘ldi.

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